

Unextendible Product Bases

Masters Thesis

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Velkommen! Mit navn er Thomas Pedersen, og jeg vil i dag tale om mit speciale, som har Titlen “Characteristics of Unextendible Product Bases”. UPBere, som forkortelsen af Unextendible Product Bases, er en ny graen af kvante informations teorien. Jeg vil i min praesentation antage at i har lidt kandskab til den kvante mekaniske model.

1 – Overview

Overview

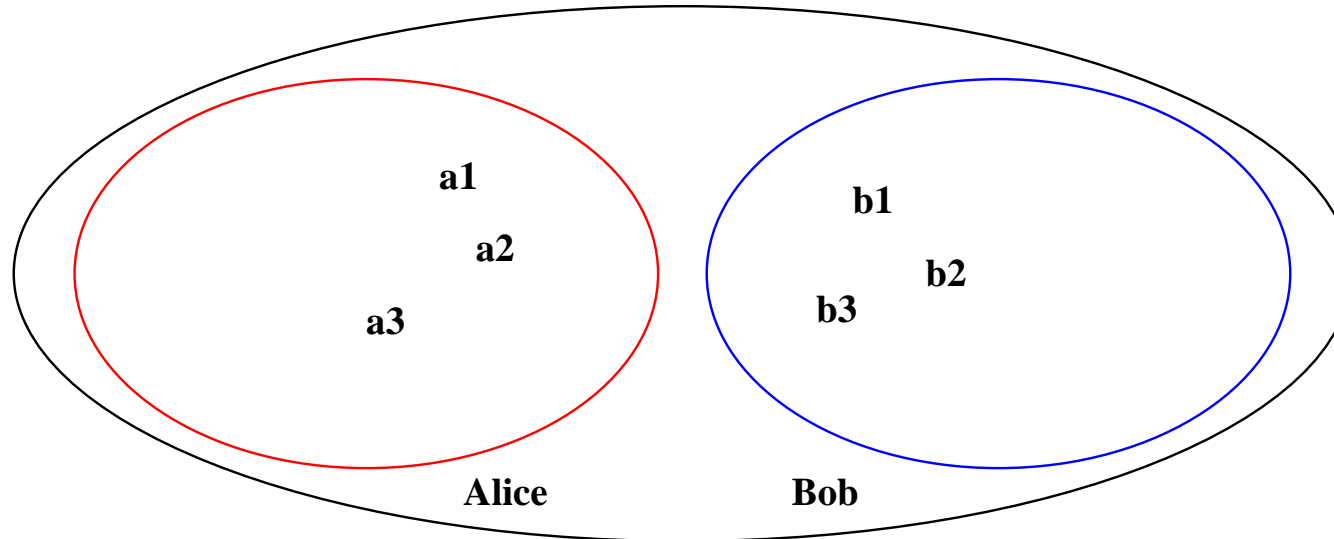
- Unextendible Product Bases
- Implications to quantum information theory and communication
- Known and new results
- Summary

1 – Overview

- Kvantemekanik

2 – Unextendible Product Bases

Unextendible Product Bases



Product basis:

$$|a_1\rangle \otimes |b_1\rangle = |a_1 b_1\rangle$$

$$|a_2\rangle \otimes |b_2\rangle = |a_2 b_2\rangle$$

$$|a_3\rangle \otimes |b_3\rangle = |a_3 b_3\rangle$$

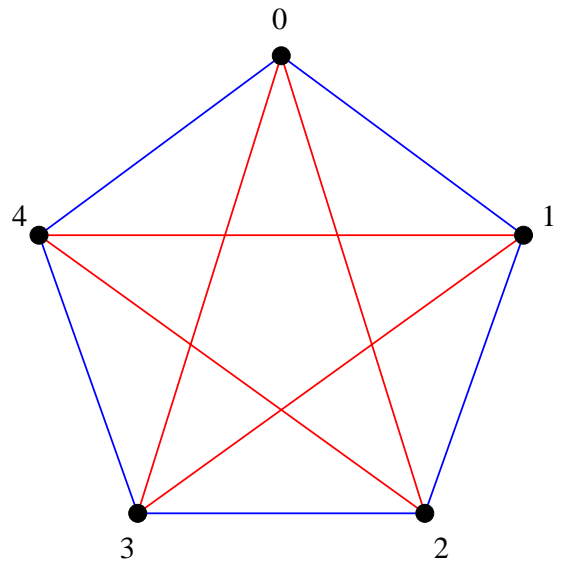
2 – Unextendible Product Bases

Jeg vil definere UPBere i 3 trin

- N personer
- Product State
- Entangled State (**ikke klassisk** - [EPR35])
- Product Basis
- **UPB**

2 – Unextendible Product Bases

Orthogonality Graph Representation



Orthogonality in Alice's system ————
 Orthogonality in Bob's system ————

$$|\psi_0\rangle = |0\rangle \otimes \frac{1}{\sqrt{2}}|0-1\rangle$$

$$|\psi_1\rangle = \frac{1}{\sqrt{2}}|0-1\rangle \otimes |2\rangle$$

$$|\psi_2\rangle = |2\rangle \otimes \frac{1}{\sqrt{2}}|1-2\rangle$$

$$|\psi_3\rangle = \frac{1}{\sqrt{2}}|1-2\rangle \otimes |0\rangle$$

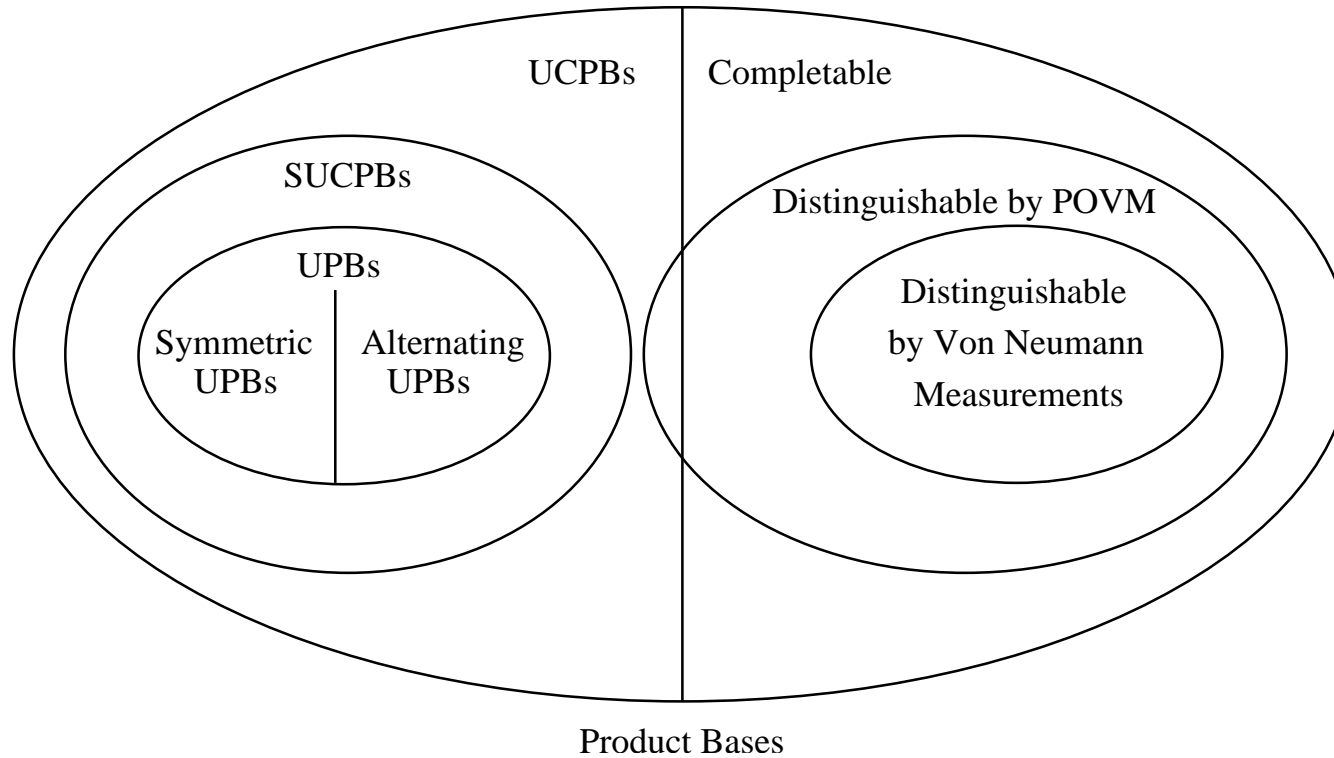
$$|\psi_4\rangle = \frac{1}{\sqrt{3}}|0+1+2\rangle \otimes \frac{1}{\sqrt{3}}|0+1+2\rangle$$

2 – Unextendible Product Bases

- Bygger paa egenskaben ved indre produkt
 $(\langle\phi_{11}|\langle\phi_{12}|)(|\phi_{21}\rangle|\phi_{22}\rangle) = \langle\phi_{11}|\phi_{21}\rangle\langle\phi_{12}|\phi_{22}\rangle$
- UPB i $\mathbb{C}^3 \otimes \mathbb{C}^3 = \mathbb{C}^9$

2 – Unextendible Product Bases

Classification of Product Bases



2 – Unextendible Product Bases

Vi har nu set PB og UPB, lad os se hvordan de er relateret til hinanden

- Distinguishability og Completeness viser sig at **relatere** til UPB

3 – Implications

Implications

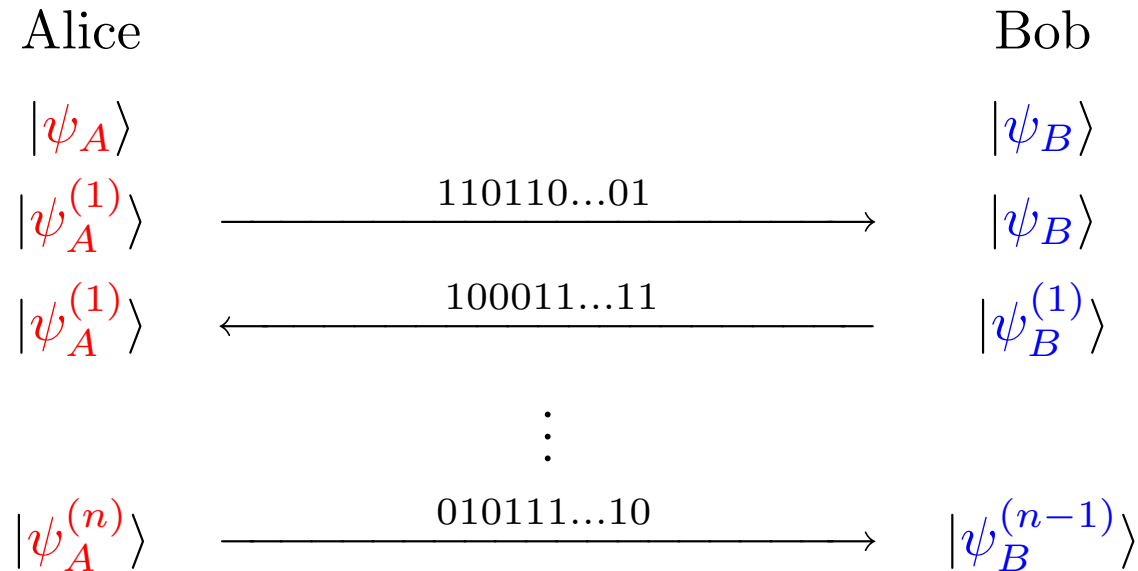
- ✓ Unextendible Product Bases
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3 – Implications



3 – Implications

Local Operations and Classical Communication



- Important for understanding non-classical quantum phenomena
- Distillation of entanglement
- Distinguishing quantum states

3 – Implications

Der hvor UPBere viser sig at være vigtige er i et scenarie kaldet **LOCC**

- Har forbundet forståelsen af Entanglement til **kendt matematik** (majorisering)
- God entanglement til Teleportering, Noegleudveksling, Error-correction, Algoritmer

3 – Implications

Bound Entanglement

- Entanglement as a resource
- “Bad news” for quantum communication
- Theoretically intriguing
- Activation of Bound Entanglement

3 – Implications

- **LOCC**
- Maximal Entanglement
- Quantum key distribution, Error-correction, Algorithms, Teleporting
- [HHH98] Horodecki, Horodecki, and Horodecki

3 – Implications

Indistinguishability

- A new form of non-classical behaviour
- “Good news” for quantum information theory?
- Quantum key exchange protocol

3 – Implications

- [BDF⁺98] Bennett, DiVincenzo, Fuchs, Mor, Shor, Smolin, Rains and Wootters
- Opdagelse taet forbundet med opdagelse af UPBere

4 – Known and New Results

Known and New Results

- ✓ Unextendible Product Bases
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4 – Known and New Results

Opsummer distill og distinguishability

4 – Known and New Results

UPBs are

- Indistinguishable
- Related to Bound Entanglement
- Strongly uncompletable

4 – Known and New Results

- UPB er indistinguishable, **seperable** og ortogonale
- Der er Bound Entanglemen til UPB, men ikke UPB til Bound Entanglement

4 – Known and New Results

Upper and Lower Bounds on Size

Lower

- A simple lower bound
- Alon-Lovász Criterion

Upper

- A lower bound on Bound Entanglement

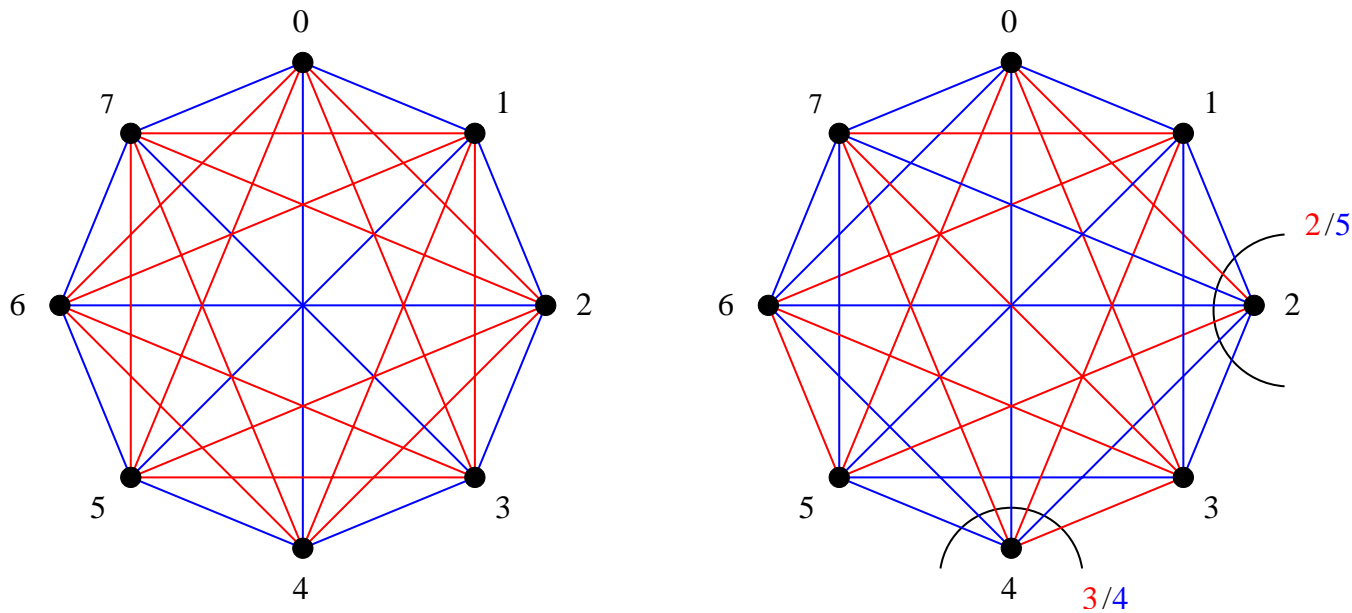
4 – Known and New Results

- $n \geq \sum_{k=1}^m (d_k - 1) + 1$ [BDM⁺98] Bennett, DiVincenzo, Mor, Shor, Smolin, and Terhal
- $\neg((m = 2 \wedge 2 \in \{d_1, d_2\}) \vee (n \text{ odd} \wedge d_k \text{ even}))$
- **Alternative bevis** for Alon-Lovász Criterion
- $n \leq d_1 d_2 - 3$ [HSTT99] Horodecki, Smolin, Terhal, and Thapliyal

4 – Known and New Results

Orthogonality Graphs

- Symmetric UPBs are exactly the trivial minimal UPBs
- Orthogonality graphs of symmetric UPBs are “nice”
- Equivalence classes of orthogonality graphs



4 – Known and New Results

- Definer **Symmetrisk og Alternierende UPB**
- Alle kanter er forbundet til præcis $(d_k - 1)$ kanter med farve k
Alle kanter har præcis een farve
- Sym i $\mathbb{C}^5 \otimes \mathbb{C}^4$, MIN Alter i $\mathbb{C}^4 \otimes \mathbb{C}^4$
- I $\mathbb{C}^3 \otimes \mathbb{C}^d$, hvor d er ulige, er alle minimale UPBere ækvivalente
- **Graf isomorfi**

5 – Summary

Summary

- ✓ Unextendible Product Bases
- ✓ Implications to quantum information theory and communication
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5 – Summary



5 – Summary

Importance of UPBs

- Understanding Entanglement is vital
- Indistinguishability is a new non-classical phenomena
- UPB is a strong mathematical model related to both

Future work

- Characteristics of Bound Entanglement related to UPBs
- UPBs and graph theory

5 – Summary

- UPBs og graf teori
- Entanglement og majorisering
- Drlig forstelse af Bound Entanglement

Tak for jeres opmaeksomhed