Course Notes for
Simulating Smoke and Water in CG
Ray Marching

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This document describes ray marching in detail. For the derivations of the equations see the slides from the lecture. Ray marching is an extension to ray tracing that enables you to compute the light (radiance) along a ray going through participating media such as moke. Recall that the radiance along a ray in non-participating media such as vacuum or (to some degree) clear air is constant. However, in the case of participating media, the radiance is decaying exponentially as it travels through the media due to scattering and absorption effects. In addition radiance is in-scattered in the direction that the ray is traveling, hereby adding more radiance in the direction of the ray. In this course we will assume that we have a non-emissive volume and that only point light sources are present in the scene.

Refering to figure 1, assume that we wish to compute how the point s on the sphere looks to the eye at point e. We construct a ray, r, going from s to e having origin in s and direction $\Theta = \overrightarrow{e-s}$. The gray box represents a block of smoke that the ray passes through. The radiance leaving s in direction Θ is given by $L(s \rightarrow \Theta)$ and the radiance arriving at e in direction $-\Theta$ is given by $L(e \leftarrow -\Theta)$ (note we have to negate the direction Θ at e to represent the *incoming* radiance). Note that in practice the radiance is a vector with a red, green and blue component respectively. In addition to the sphere, the eye and the smoke, we assume there are N point light sources in the scene. In the case of figure 1, $N = 1$. A point light source is light emanating in all directions from a single point in space. It can be situated anywhere in the scene, even inside the smoke. The *intensity* of a point light source that shines equally bright in all directions is given by $I = \frac{\Phi}{4\pi}$, where I is the intensity and Φ is the so-called *flux*. The flux is simply the total power, or energy per time, emanating from the light source. It is measured in Watts like a normal light bulb that you all know. So in order to specify the intensity of a light source the only parameter you have to adjust is the flux; The higher the flux, the brighter the light source.
To compute the radiance arriving at e in direction $-\Theta$, $L(e \leftarrow -\Theta)$, from the

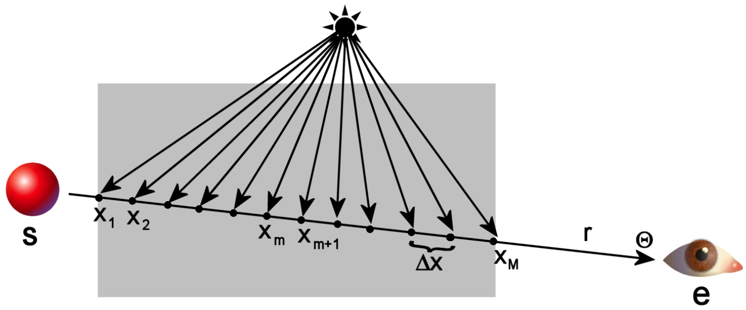

Figure 1: Ray Marching from eye to sphere.

radiance emanating at s in direction Θ, we utilize a recursive formula which is known as ray marching. Ray marching utilizes numerical integration. In order to do this, we divide the ray from s to e into M small segments each of length Δx, and number the endpoints of the segments consequtively starting from where the ray enters the participating media, see figure 1. Now assume that we are at point x_{m+1} and wish to compute the radiance leaving x_{m+1} in direction Θ. The recursive formula for this is given by

$$
\begin{aligned}
L(x_{m+1} \to \Theta) \quad = \quad & \\
& \exp(-\sigma_t(x_{m+1})\Delta x) L(x_m \to \Theta) + \\
& \sum_{n=1}^{N} L_n(x_{m+1} \leftarrow \Psi_n) p(x_{m+1}, \Psi_n \leftrightarrow \Theta) \alpha(x_{m+1}) \sigma_t(x_{m+1}) \Delta x
\end{aligned}
$$

where σ_t is the extinction coefficient, Δx is the step size, $L(x_m \to \Theta)$ is the recursive call to the previous point on the ray, N is the number of light sources, $L_n(x_{m+1} \leftarrow \Psi_n)$ is the radiance coming from the nth point light source in direction Ψ_n, $p(x_{m+1}, \Psi_n \leftrightarrow \Theta)$ is the phase function determining how much of the in-coming radiance from direction, Ψ_n scatters into the out-going direction along the ray, Θ, and finally α is the albedo. Note that in an implementation there is no need to compute the second term of the sum as well as the factor of exponential decay in the first term in the case where $\sigma_t = 0$ (corresponding to a voxel with no smoke).

To give you an intuition about the effect of the exponential falloff, figure 2 depicts the function $\exp(-\sigma_t \cdot x)$ for four different values of σ_t.

To get started with the above formula, recall that the extinction coefficient is simply proportional to the smoke density, $\rho(x_{m+1})$, so you could start by setting $\sigma_t(x_{m+1}) = \rho(x_{m+1})$. Also you can just start by setting the albedo equal to 1 at all points, so $\alpha(x_{m+1}) = 1$. In addition you can start by assuming that the radiance coming from light sources is not affected by the participating media,

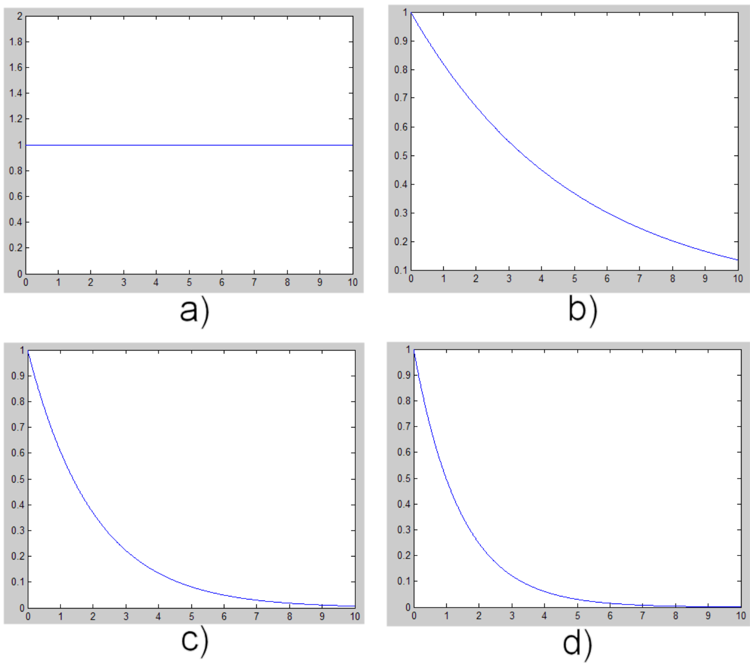

Figure 2: Illustration of the exponential falloff. The graphs show the function $\exp(-\sigma_t \cdot x)$ for four different values of σ_t. a) $\sigma_t = 0$. b) $\sigma_t = 0.2$. c) $\sigma_t = 0.5$. d) $\sigma_t = 0.7$.

hence setting $L_n(x_{m+1} \leftarrow \Psi_n) = \frac{I_n}{|l_n - x_{m+1}|^2}$, where I_n is the intensity of and l_n is the position of the nth point light source. Finally, start with a single point light source and the constant isotropic phase function, $p(x_{m+1}, \Psi_n \leftrightarrow \Theta) = \frac{1}{4\pi}$. With these simplifications, suddenly the above equation looks a lot simpler:

$$\begin{aligned} L(x_{m+1} \rightarrow \Theta) &= \\ &\exp(-\rho(x_{m+1})\Delta x) L(x_m \rightarrow \Theta) + \\ &\frac{I_1 \rho(x_{m+1})}{4\pi |l_1 - x_{m+1}|^2} \Delta x \end{aligned}$$

Computing the radiance $L(x_M \rightarrow \Theta)$ is initiated by a call to the above recursive formulas that then return the proper radiance. When evaluating $L(x_1 \rightarrow \Theta)$, the recursive call to $L(x_0 \rightarrow \Theta)$ simply equals $L(s \rightarrow \Theta)$ and the recursion stops. Note that the radiance $L(x_M \rightarrow \Theta)$ can also easily be computed iteratively by instead starting from x_1 and moving forward. Finally note that $L(x_M \rightarrow \Theta) = L(e \leftarrow -\Theta)$.

The only thing left out in the above discussion is how to properly compute $L_n(x_{m+1} \leftarrow \Psi_n)$, the radiance from the nth point light source. This is needed in order to get the cool looking self-shadowing effects in the smoke. It is rather simple, but we have to remember that the radiance from the point light source

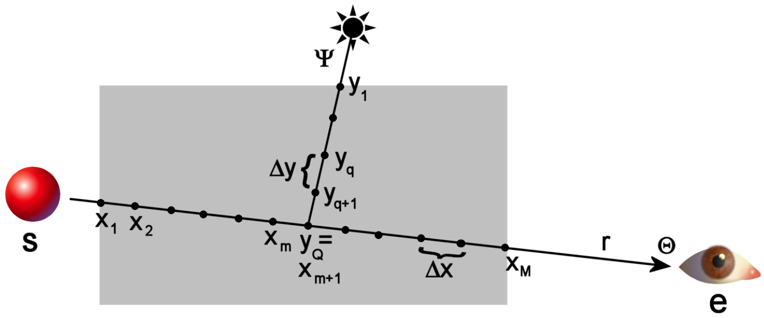

Figure 3: Ray Marching from x_{m+1} to point light source.

is also affected by the participating media on its way to the point x_{m+1}. Again we have to use ray marching, but since we only consider in-scattered light at x_{m+1} due to direct illumination, this time we will ignore the in-scattering term on the radiance from the light source to the point x_{m+1}. In figure 3 we have divided the ray between the point x_{m+1} and the position of the light source, l_1, into Q segments, each of length Δy. The situation looks very simular to figure 1. Note that the point x_{m+1} equals the point y_Q. Again we can formulate the computation of the radiance recursively:

$$L(y_{q+1} \to -\Psi_n) = \exp(-\sigma_t(y_{q+1})\Delta y) L(y_q \leftarrow -\Psi_n)$$

Finally note that $L_n(x_{m+1} \leftarrow \Psi_n) = L(y_Q \to -\Psi_n)$ and in addition you should set the boundary condition, $L(y_1 \leftarrow -\Psi_n)$ equal to $\frac{I_n}{|l_n - x_{m+1}|^2}$.