SYNTHESIS OF P/T-NETS BASED ON REGIONS POTENTIAL APPLICATIONS

SUMMARY OF JOINT WORK

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1995 ...

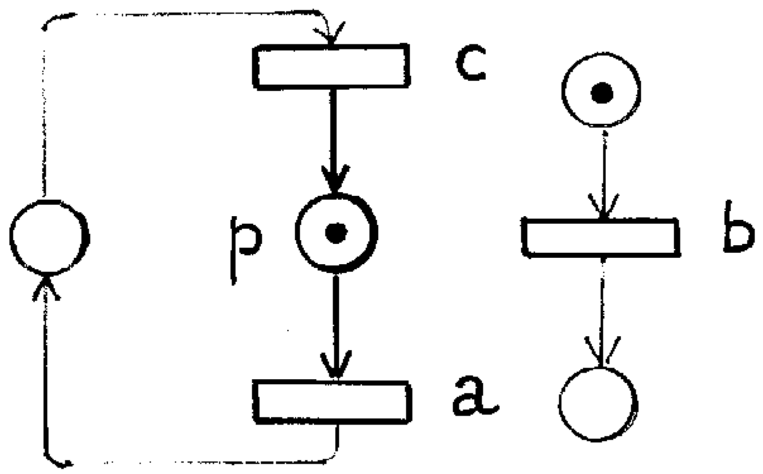
BASIC PROBLEMS

- GIVEN A TRANSITION SYSTEM $\mathcal{T} = (Q, E, T, q_0)$ DECIDE WHETHER EXISTS & CONSTRUCT $\mathcal{N} = (P, E, F, M_0)$ SUCH THAT $\mathcal{T} \cong SG(\mathcal{N})$

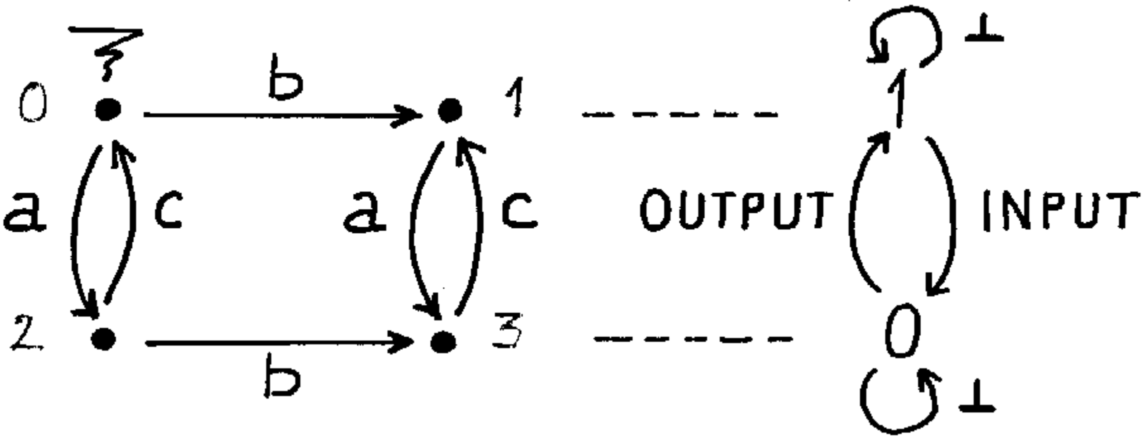
- GIVEN A LANGUAGE $\mathcal{L} \subseteq E^*$ DECIDE WHETHER EXISTS & CONSTRUCT $\mathcal{N} = (P, E, F, M_0)$ SUCH THAT $\mathcal{L} = L(\mathcal{N})$

EVENTS OF NETS LABELLED INJECTIVELY

REGIONS OF ELEMENTARY NETS



$\mathcal{C} = SG(\mathcal{N})$ τ



PLACE $\leftrightarrow$ REGION $(\sigma, \eta) : \mathcal{C} \to \tau$

$\sigma 0 = \sigma 1 = 1$	$\sigma 2 = \sigma 3 = 0$
$\eta a =$ OUTPUT	$\eta c =$ INPUT

EHRENFEUCHT-ROZENBERG AXIOMS

GIVEN $\mathcal{T} = (Q, E, T, q_0)$

LET $\mathcal{T}^* = (P, E, F, M_0)$

WITH $P = \mathrm{Hom}(\mathcal{T}, \tau)$

$p = (\sigma, \eta) : \mathcal{T} \to \tau$ REGION

$M_0(p) = \sigma q_0 \in \{0, 1\}$

$F(p, e) = \eta e \in \{\text{INPUT}, \text{OUTPUT}, \perp\}$

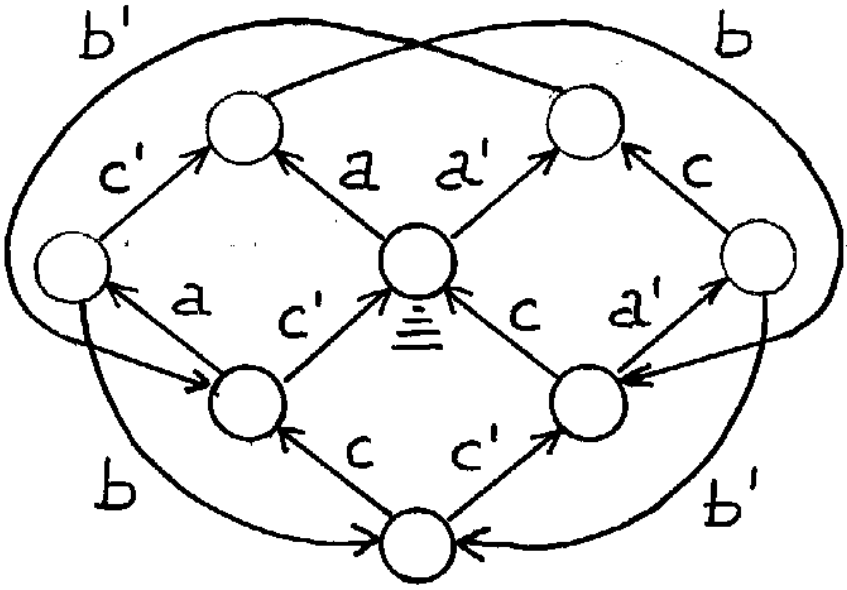
$\mathcal{T} \cong SG(\mathcal{T}^*)$ IFF SEPARATED

SSA $q \neq q' \Rightarrow \exists (\sigma, \eta) \ \sigma q \neq \sigma q'$

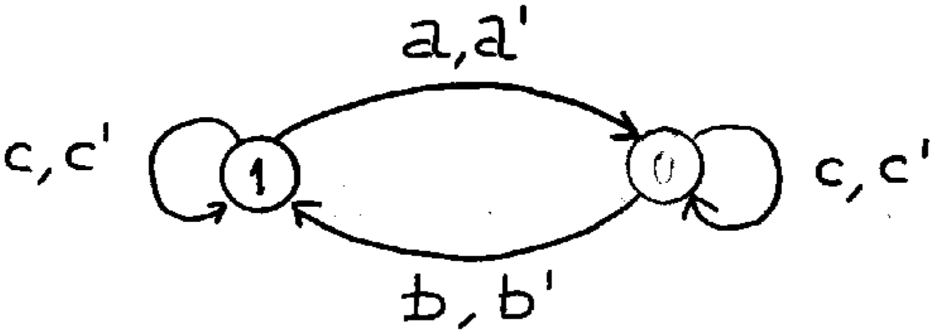
ESSA $q \overset{e}{\not\to} \Rightarrow \exists (\sigma, \eta) \ \sigma q \overset{\eta e}{\not\longrightarrow}$

$\sigma q = 0, \eta e = \text{INPUT} \vee \sigma q = 1, \eta e = \text{OUTPUT}$

DECIDABLE & CONSTRUCTIVE !

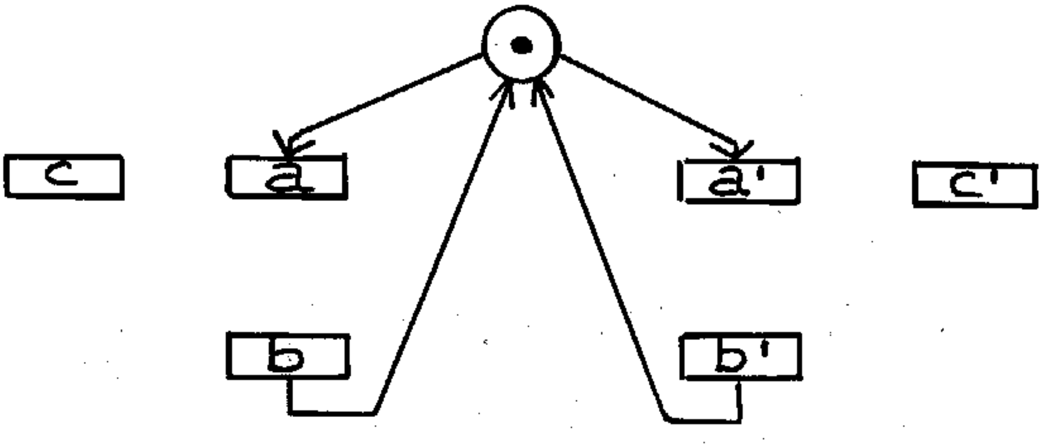


$\mathcal{G}$



τ

EXPLAINS WHY $q_0 \not\xrightarrow{b}$



FOR FINITE TRANSITION SYSTEMS

SYNTHESIS OF ELEMENTARY NETS

NP-COMPLETE

BUT EFFICIENT TOOL PETRIFY

(EHRENFEUCHT-ROZENBERG REGIONS)

SYNTHESIS OF P/T-NETS

POLYNOMIAL

LESS EFFICIENT TOOL SYNET

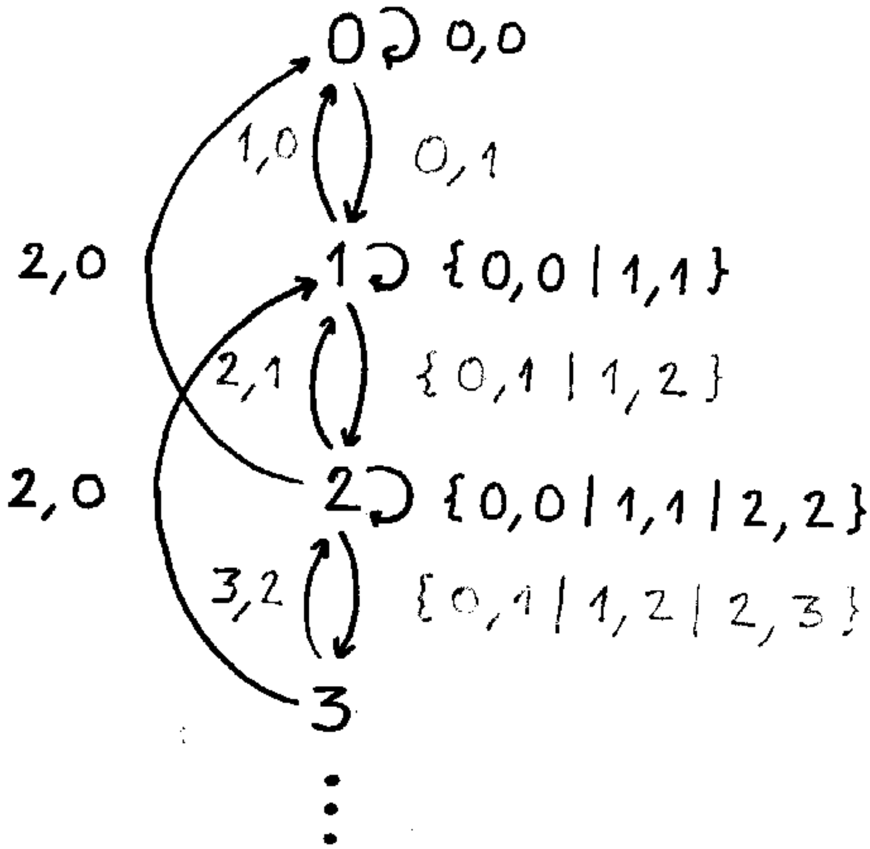
(MUKUND REGIONS)

DIFFERENT RANGES OF APPLICATION

PETRIFY CIRCUITS

SYNET DISTRIBUTED PROTOCOLS/CONTROL

τ FOR P/T-NETS



$\tau = (\mathbb{N}, \mathbb{N} \times \mathbb{N}, \longrightarrow)$ WHERE

$i \xrightarrow{n,m} j$ IFF

$i \geq n \wedge j = i - n + m$

SYNTHESIS OF P/T-NETS

GIVEN $\mathcal{C} = (Q, E, T, q_o)$

LET $\mathcal{C}^* = (P, E, F, M_o)$

WITH $P = \mathrm{Hom}(\mathcal{C}, \tau)$

$p = (\sigma, \eta) : \mathcal{C} \to \tau$ REGION

$M_o(p) = \sigma q_o \in \mathbb{N}$

$F(p, e) = (p^\bullet e, e^\bullet p) = \eta e$

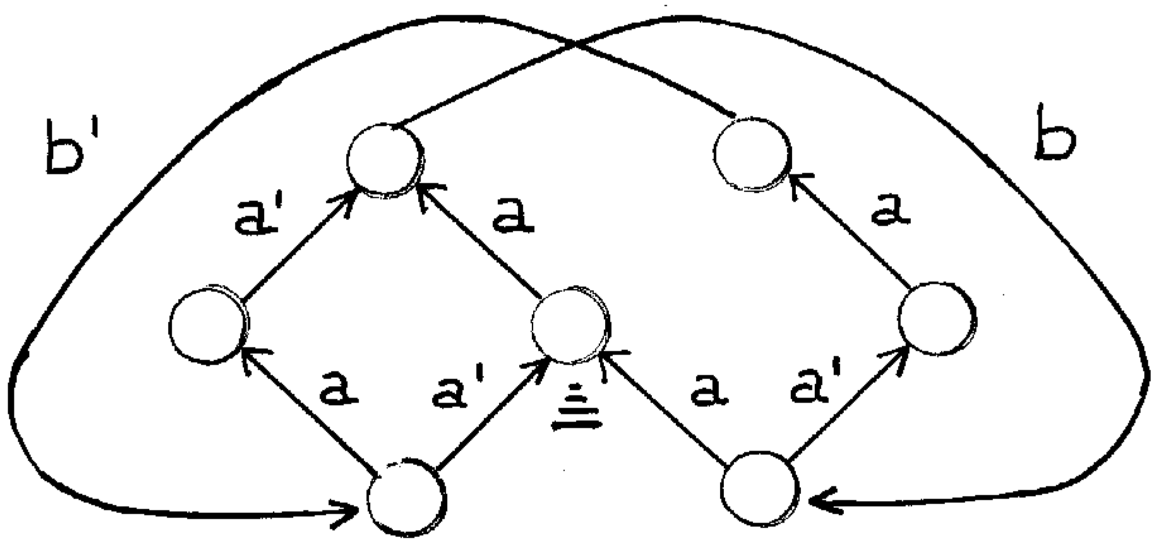
$\mathcal{C} \cong SG(\mathcal{C}^*)$ IFF SEPARATED

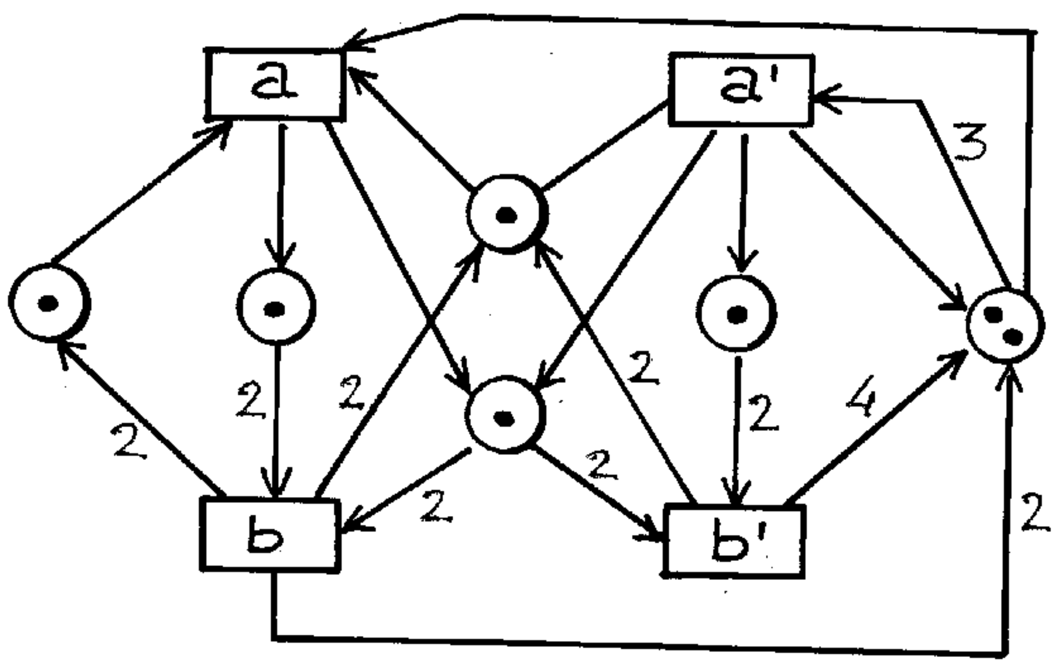
SSA $q \neq q' \Rightarrow \exists (\sigma, \eta)\ \sigma q \neq \sigma q'$

ESSA $q \stackrel{e}{\not\to} \Rightarrow \exists (\sigma, \eta)\ \sigma q \stackrel{\eta e}{\not\to}$

$\sigma q = i, \ \eta e = (n, m), \ i < n$

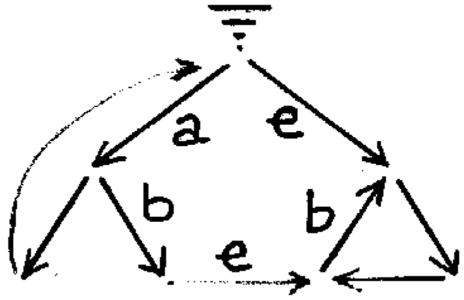
DECIDABLE? ω REGIONS!





LINEAR PROGRAMMING > INTUITION !

TRANS. SYSTEM = SPAN. TREE + CHORDS



CHORDS $\rightarrow$ FUNDAMENTAL CYCLES

$\gamma = abebe^{-} \quad \vec{\gamma}a = 1 \quad \vec{\gamma}b = 2 \quad \vec{\gamma}e = 0$

$$p = (\overbrace{M_0(p)}^{\sigma q_0}, \overbrace{p^\bullet e_1, e_1^\bullet p, \cdots, p^\bullet e_n, e_n^\bullet p}^{\eta})$$

IS A REGION IFF

- $\Sigma_i \; \vec{\gamma} e_i \times (e_i^\bullet p - p^\bullet e_i) = 0$

  FOR EVERY FUNDAMENTAL CYCLE

- $M_0(p) + \Sigma_i \; \vec{\beta} e_i \times (e_i^\bullet p - p^\bullet e_i)$

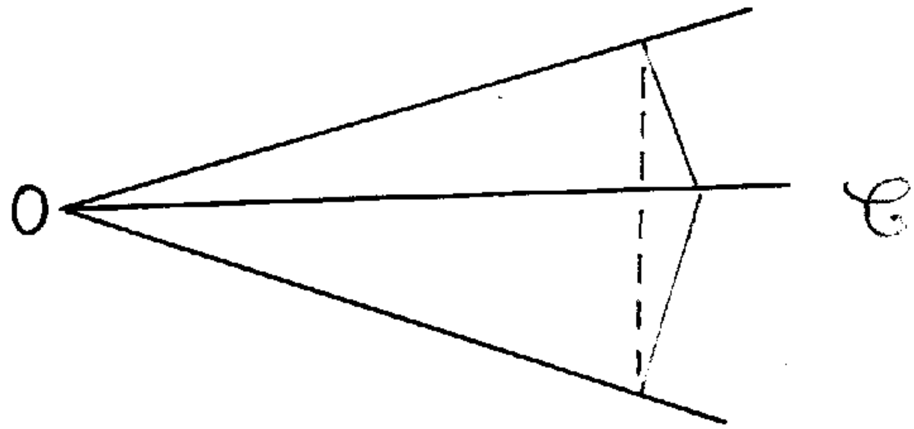
  $- p^\bullet e \geq 0$

  FOR EVERY BRANCH β LEADING

  TO STATE q SUCH THAT $q \xrightarrow{e}$

FINITE NB. OF HOMOGENOUS LIN. INEQ.

$\Rightarrow$ REGIONS FORM A POLYHEDRAL CONE



EXTREMAL RAYS COMPUTED WITH

CHERNIKOVA's ALGORITHM

SSA $\exists p \in \mathcal{C} \quad \langle \psi, p \rangle \neq 0$

ESSA $\exists p \in \mathcal{C} \quad \langle \psi, p \rangle < 0$

CHECK ON EXTREMAL RAYS

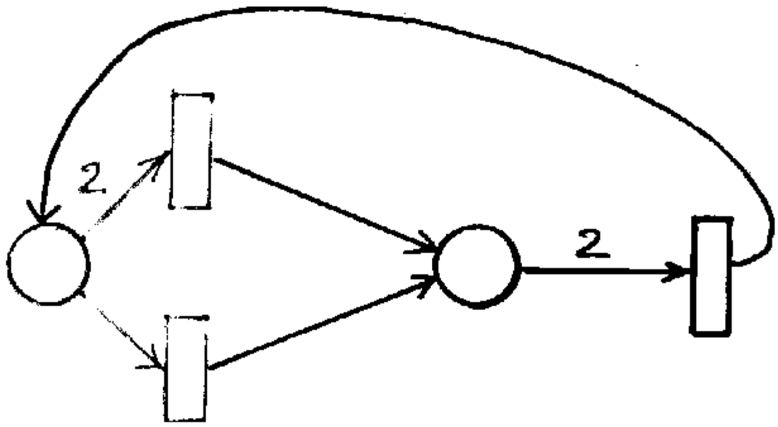
DECIDABLE & CONSTRUCTIVE

OK FOR INFINITE CONTEXT FREE GRAPHS

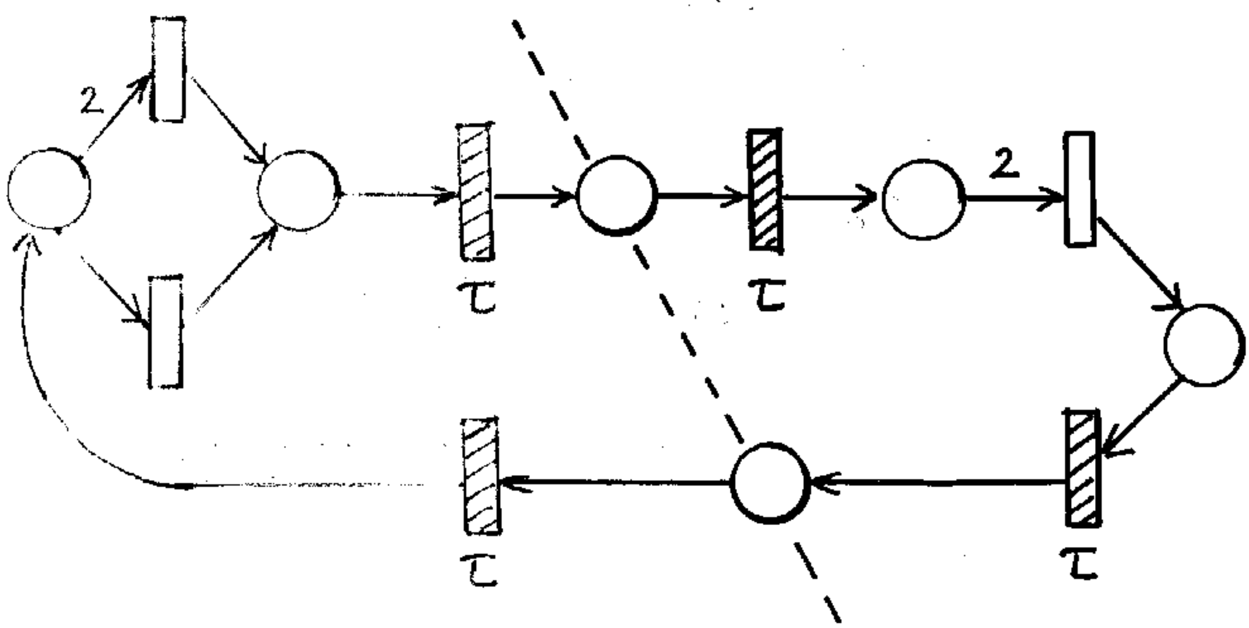
DISTRIBUTABLE PN's

EVENTS LOCATED ON SITES

CAN'T TAKE DISTANT RESOURCES

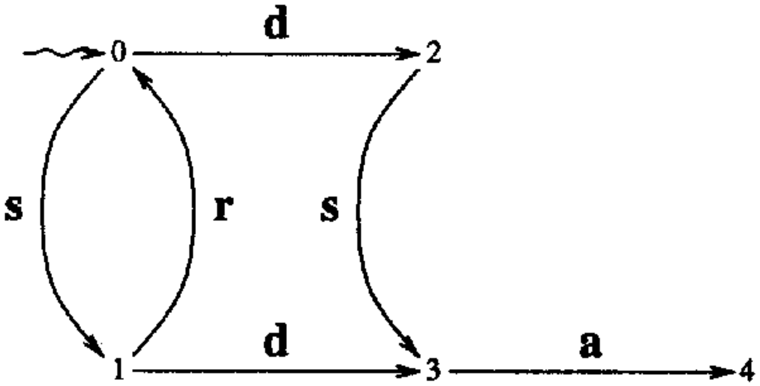


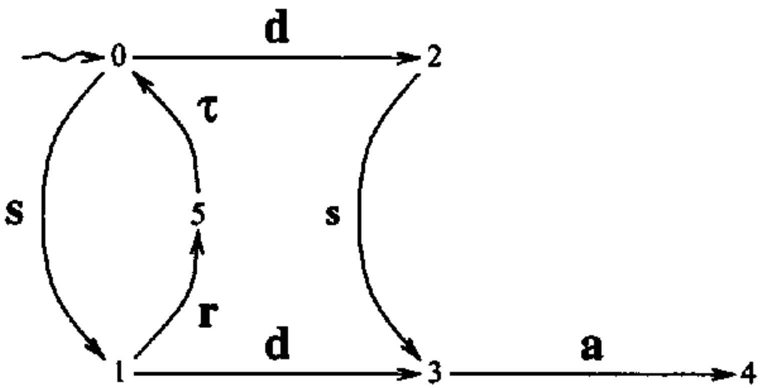
BRANCHING BISIMILAR TO

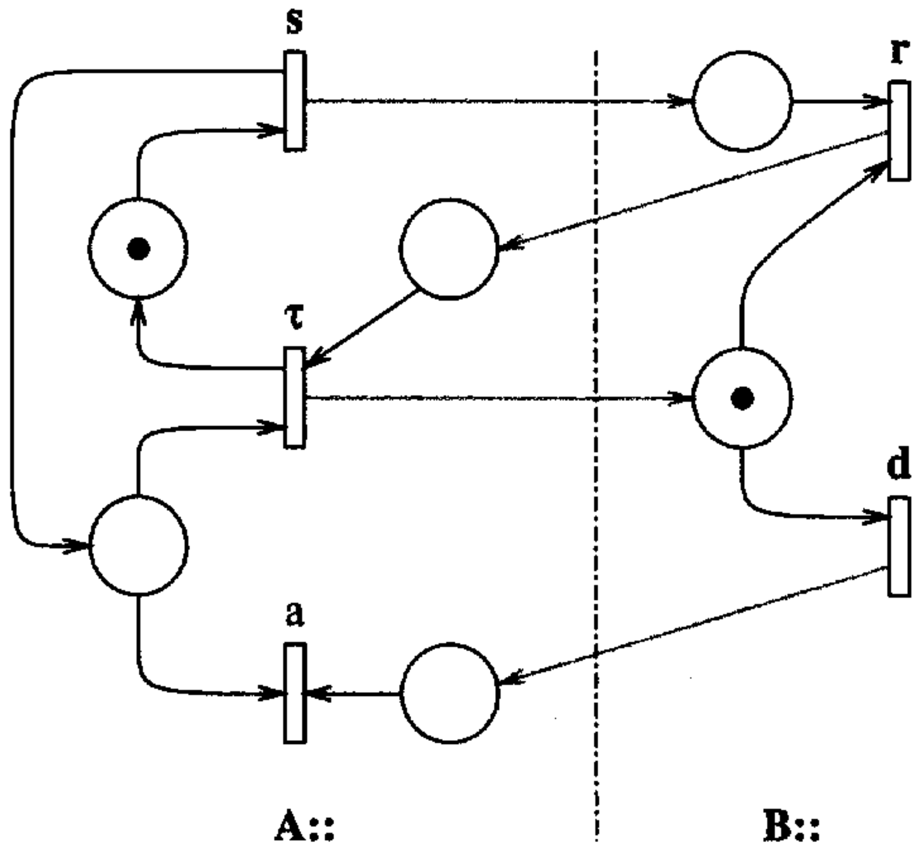


CUT AND COMPUTE SEPARATE AUTO.

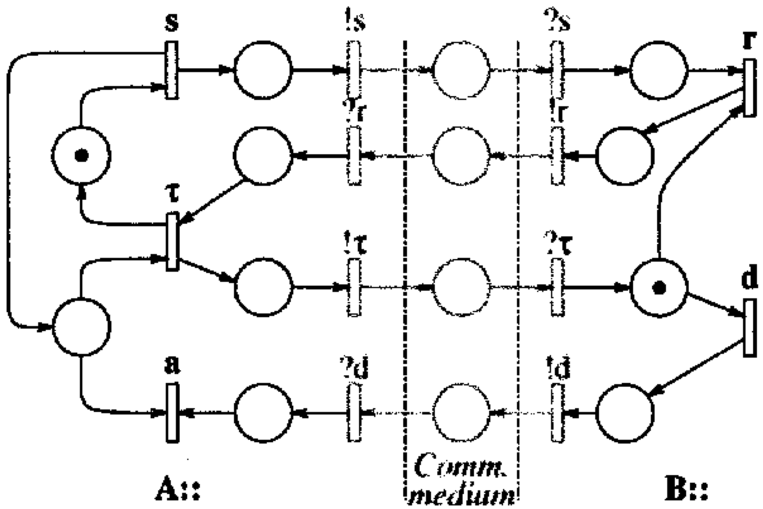
The Simplified INRES Protocol

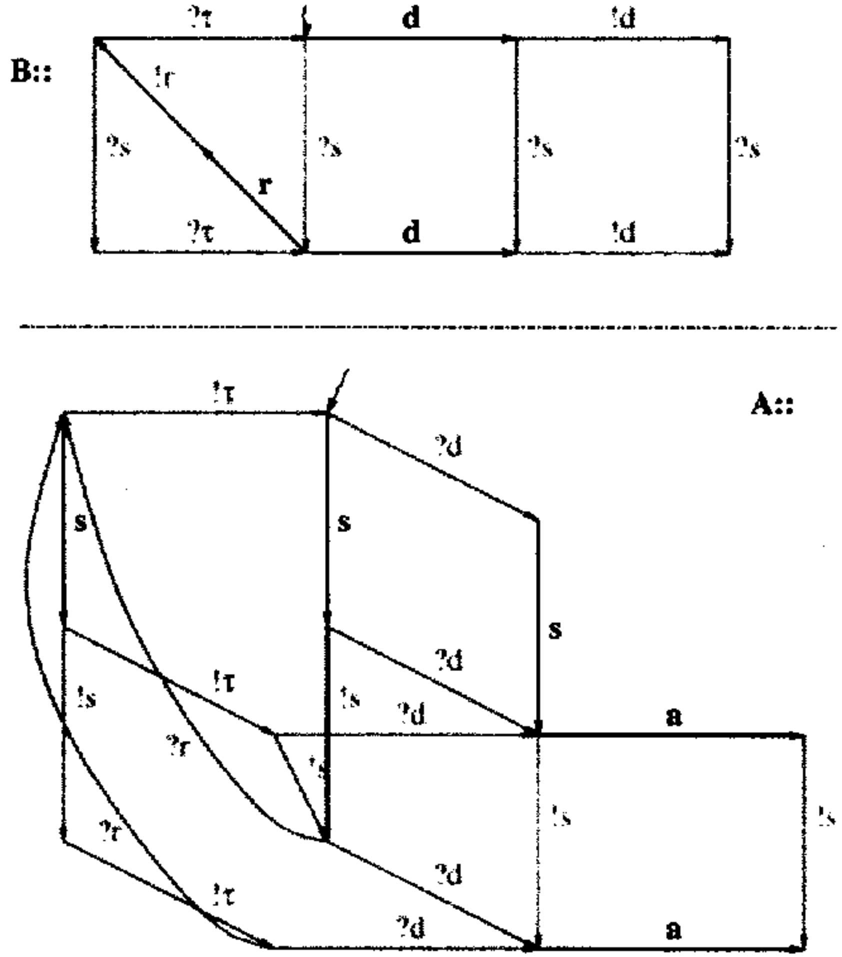






Towards Communicating Automata





REGIONS OF PREFIX CLOSED LANGUAGES

$\mathcal{L} \subseteq E^*$ $w = u.v \in \mathcal{L} \Rightarrow u \in \mathcal{L}$

PARIKH IMAGE $\vec{u} : E \longrightarrow \mathbb{N}$

$\vec{u}(e)$ COUNTS OCCURRENCES e IN u

$p = (M_0(p), p^\bullet e_1, e_1^\bullet p, \cdots, p^\bullet e_n, e_n^\bullet p)$

REGION OF $\mathcal{L}$ IFF $\forall ue \in \mathcal{L}$

$M_0(p) + \Sigma_i \ \vec{u}(e_i) \times (e_i^\bullet p - p^\bullet e_i)$

$- p^\bullet e \geqslant 0$

$\mathcal{L}^*$ NET WITH ALL REGIONS AS PLACES

$\mathcal{L} = L(\mathcal{L}^*)$ IFF $\forall u \in \mathcal{L}$ $ue \notin \mathcal{L} \Rightarrow$

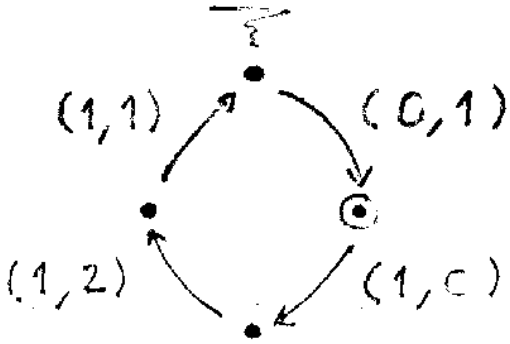
$\exists p \in$ REGIONS :

$M_0(p) + \Sigma_i \ \vec{u}(e_i) \times (e_i^\bullet p - p^\bullet e_i) - p^\bullet e < 0$

PN CLOSURE OF $\mathcal{L}$

SUPPOSE $\forall e \in E$

$\downarrow e = \{ \vec{u} \mid u e \in \mathcal{L} \}$ SEMILINEAR



$$\downarrow e = \bigcup_{i=1}^{n} S_i \cdot T_i^* \quad S_i, T_i \in \mathcal{P}_{fin}(\mathbb{N}^E)$$

THEN p REGION OF $\mathcal{L}$ IFF $\forall e$

- $\forall i \; \forall s \in S_i$

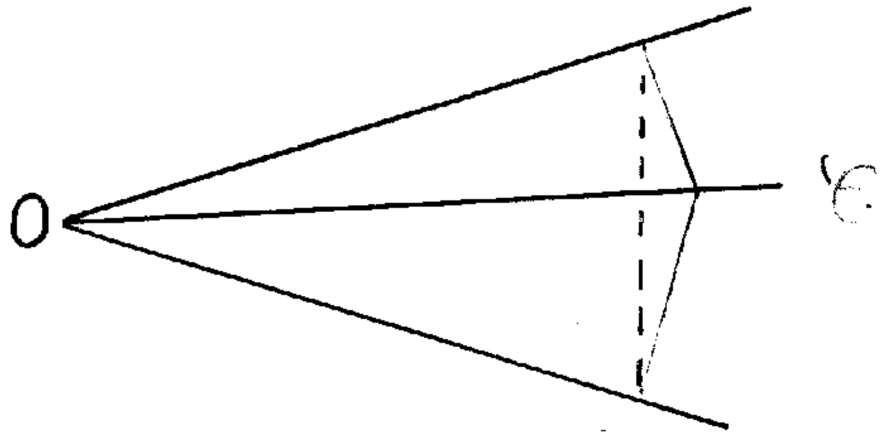
$$M_0(p) + \sum_j s(e_j) \times ({e_j}^\bullet p - p^\bullet e_j) - p^\bullet e \geqslant 0$$

- $\forall i \; \forall t \in T_i$

$$\sum_j t(e_j) \times ({e_j}^\bullet p - p^\bullet e_j) \geqslant 0$$

FINITE SYSTEM OF LIN. INEQUALITIES

$\Longrightarrow$ REGIONS FORM A POLYHEDRAL CONE



$N(\mathcal{L})$ NET WITH EXTREMAL RAYS

AS PLACES $p = (M_0(p), p^\bullet e_1, e_1^\bullet p \, \dots)$

THEN $\mathcal{L} \subseteq L(N(\mathcal{L}))$ & $\forall N'$:

$\mathcal{L} \subseteq L(N') \Rightarrow L(N(\mathcal{L})) \subseteq L(N')$

CLOSURE / BEST APPROXIMATION

MAY BE APPLIED TO:

REGULAR OR CONTEXT-FREE LANG.

HMSC LANGUAGES (MOVEP'2K)

EXACT REALIZATION

SUPPOSE FURTHER $\forall e \in E$

$$\uparrow e = \{\vec{u} \mid u \in \mathcal{L} \wedge ue \notin \mathcal{L}\}$$

SEMILINEAR

$$\uparrow e = \bigcup_{i=1}^{n} S_i \cdot T_i^* \quad S_i, T_i \subseteq \mathbb{N}^E$$

THEN $\mathcal{L} = L(N(\mathcal{L}))$ IFF $\forall e$

$\forall i \ \forall s \in S_i \ \exists p \in N(\mathcal{L})$:

$$M_0(p) + \sum_j s(e_j) \times ({e_j}^\bullet p - {}^\bullet p\, e_j) - {}^\bullet p\, e < 0$$

S_i FINITE $\Rightarrow$ DECIDABLE

MAY BE APPLIED TO:

REGULAR / DETERMINISTIC CONTEXT-FREE

UNDECIDABLE FOR:

CONTEXT-FREE / HMSC LANGUAGES !

DISTRIBUTED CONTROL SYNTHESIS

DISTRIBUTED PLANT

$\mathcal{N}_p = (P, E, F, M_o) \qquad \lambda : E \to \{1 \ldots n\}$

CONTROL OBJECTIVE (TOLERANCE)

$\mathcal{L}_1 \subseteq L(\mathcal{N}_p \times \mathcal{N}_c) \subseteq \mathcal{L}_2$

DISTRIBUTED SUPERVISOR

$\mathcal{N}_c = (P', E, F', M'_o)$

$p'^{\bullet} e \neq 0 \Rightarrow \lambda' p' = \lambda e \in \{1 \ldots n\}$

UNCONTROLABLE EVENTS

$\lambda' p' = j \Rightarrow p'^{\bullet} e = 0$

UNOBSERVABLE EVENTS

$\lambda' p' = j \Rightarrow p'^{\bullet} e = e^{\bullet} p'$

SET $\mathcal{N}_c = \mathcal{L}_1^{*}$

CHECK $L(\mathcal{N}_p) \cap L(\mathcal{N}_c) \subseteq \mathcal{L}_2$

SHORTCOMINGS

SYMBOLIC SYNTHESIS

SYNTHESIS OF LIVE NETS

SYNTHESIS UP TO SILENT ACTIONS

CASE OF NON SEMILINEAR LANGUAGES

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