

A Group-theoretical Approach to Petri Nets

Bruno Brosowski*
Universität Frankfurt am Main

In the talk we present a unified approach to different types of Petri nets such as ST-nets, coloured nets, FIFO-nets, LIFO-nets, continuous nets and mixtures of these nets. This approach uses the concept of cone groups. These are groups which contain a cone compatible with the group structure. Such a cone defines a partial order on the group resp. also a right or left partial order in the case of a non-abelian group.

In the frame of these remarks and taking into account that an enabled transition can fire in different modes the following definition of a Petri net is introduced:

Definition. A Petri net with firing modes is a system $\mathcal{P} = (\mathcal{C}, T, \mathcal{M}, \text{Pre}, \text{Post})$ consisting of

- a cone group $\mathcal{C} := (G, *, e, K)$, where K is a cone contained in the group G ,
- a set T , called the set of transitions,
- a set \mathcal{M} of firing modes,
- mappings $\text{Pre} : T \times \mathcal{M} \rightarrow K \times K$ and $\text{Post} : T \times \mathcal{M} \rightarrow K \times K$

If the cone group $\mathcal{C} = (G, *, e, K)$ of a Petri net is the direct product of the cone groups $\mathcal{C}_s = (G_s, e_s, *_s, K_s)$, i.e.

$$\mathcal{C} := \prod_{s \in S} \mathcal{C}_s := \left(\prod_{s \in S} (G_s, *_s, e_s), *, e, \prod_{s \in S} K_s \right)$$

then the index set S is called a set of places of the Petri net.

Definition. A transition $t \in T$ of a Petri net \mathcal{P} is enabled in the mode $\mu \in \mathcal{M}$ with respect to the marking $m \in K$ if

$$[\pi_1 \circ \text{Pre}(t, \mu)]^{-1} * m + [\pi_2 \circ \text{Pre}(t, \mu)]^{-1} \in K \text{ or } [\pi_1 \circ \text{Pre}(t, \mu)]^{-1} * m * [\pi_2 \circ \text{Pre}(t, \mu)]^{-1} \geq_K e.$$

An enabled transition can fire. If an enabled transition $t \in T$ fires in the mode $\mu \in \mathcal{M}$ then the enabling marking $m \in K$ is transformed into the group element

$$\delta(m, t, \mu) := [\pi_1 \circ \text{Post}(t, \mu)] * [\pi_1 \circ \text{Pre}(t, \mu)]^{-1} * m * [\pi_2 \circ \text{Pre}(t, \mu)]^{-1} * [\pi_2 \circ \text{Post}(t, \mu)],$$

which is contained in the cone K , i.e. $\delta(m, t, \mu)$ is again a marking.

For special choices of the PN-parameters one obtains the different types of Petri nets. Some examples:

Petri net	Cone group	Firing modes	Pre / Post
ST – nets, N places	$\mathcal{C} = (\mathbb{Z}^N, +, 0, \mathbb{N}_0^N)$	$\# \mathcal{M} = 1$	$\forall_{t \in T} \text{Pre}(t), \text{Post}(t) \in \mathbb{N}_0^{2N}$
Coloured Petri nets N places, k colours	$\mathcal{C} = (Q^N, *, 1_N, \mathbb{N}^N)$ Q pos. rat. numbers	$\mathcal{M} = \{p_1, p_2, \dots, p_k\}$ p_κ suitable primes	$\forall_{t \in T} \forall_{\kappa=1}^k$ $\text{Pre}(t, p_\kappa), \text{Post}(t, p_\kappa) \in \mathbb{N}^{2N}$
FIFO nets with N places, alphabet A	$\mathcal{C} := (F_A^N, \cdot, \varepsilon, (A^*)^N)$ F_A free group gener. by A	$\# \mathcal{M} = 1$	$\forall_{t \in T} \text{Pre}(t) \in A^* \times \{\varepsilon\}$ & $\text{Post}(t) \in \{\varepsilon\} \times A^*$
Continuous nets	$\mathcal{C} = (\mathbb{R}^N, +, 0, \mathbb{R}_0^N)$	$\mathcal{M} = \mathbb{R} \setminus \{0\}$	$\forall_{t \in T} \text{Pre}(t, \mu) = \mu \cdot \text{Pre}'(t)$ & $\text{Post}(t, \mu) = \mu \cdot \text{Post}'(t)$ $\forall_{t \in T} \text{Pre}'(t), \text{Post}'(t) \in \mathbb{R}^{2N}$

In the talk the following items will be considered in more detail:

- Homomorphisms of Petri nets, sub-Petri nets,
- Petri nets with an abelian cone group,
- S-invariants, T-invariants,
- Petri nets with sides-conditions.

*e-mail: BBrosowski@t-online.de