

# A Group-theoretical Approach to Petri Nets

Bruno Brosowski\*  
 Universität Frankfurt am Main

In the talk we present a unified approach to different types of Petri nets such as ST-nets, coloured nets, FIFO-nets, LIFO-nets, continuous nets and mixtures of these nets. This approach uses the concept of cone groups. These are groups which contain a cone compatible with the group structure. Such a cone defines a partial order on the group resp. also a right or left partial order in the case of a non-abelian group.

In the frame of these remarks and taking into account that an enabled transition can fire in different modes the following definition of a Petri net is introduced:

**Definition.** A Petri net with firing modes is a system  $\mathcal{P} = (\mathcal{C}, T, \mathcal{M}, \text{Pre}, \text{Post})$  consisting of

- a cone group  $\mathcal{C} := (G, *, e, K)$ , where  $K$  is a cone contained in the group  $G$ ,
- a set  $T$ , called the set of transitions,
- a set  $\mathcal{M}$  of firing modes,
- mappings  $\text{Pre} : T \times \mathcal{M} \rightarrow K \times K$  and  $\text{Post} : T \times \mathcal{M} \rightarrow K \times K$

If the cone group  $\mathcal{C} = (G, *, e, K)$  of a Petri net is the direct product of the cone groups  $\mathcal{C}_s = (G_s, e_s, *_s, K_s)$ , i.e.

$$\mathcal{C} := \prod_{s \in S} \mathcal{C}_s := \left( \prod_{s \in S} (G_s, *_s, e_s), *, e, \prod_{s \in S} K_s \right)$$

then the index set  $S$  is called a set of places of the Petri net.

**Definition.** A transition  $t \in T$  of a Petri net  $\mathcal{P}$  is enabled in the mode  $\mu \in \mathcal{M}$  with respect to the marking  $m \in K$  if

$$[\pi_1 \circ \text{Pre}(t, \mu)]^{-1} * m + [\pi_2 \circ \text{Pre}(t, \mu)]^{-1} \in K \text{ or } [\pi_1 \circ \text{Pre}(t, \mu)]^{-1} * m * [\pi_2 \circ \text{Pre}(t, \mu)]^{-1} \geq_K e.$$

An enabled transition can fire. If an enabled transition  $t \in T$  fires in the mode  $\mu \in \mathcal{M}$  then the enabling marking  $m \in K$  is transformed into the group element

$$\delta(m, t, \mu) := [\pi_1 \circ \text{Post}(t, \mu)] * [\pi_1 \circ \text{Pre}(t, \mu)]^{-1} * m * [\pi_2 \circ \text{Pre}(t, \mu)]^{-1} * [\pi_2 \circ \text{Post}(t, \mu)],$$

which is contained in the cone  $K$ , i.e.  $\delta(m, t, \mu)$  is again a marking.

For special choices of the PN-parameters one obtains the different types of Petri nets. Some examples:

Petri net	Cone group	Firing modes	Pre / Post
ST – nets, $N$ places	$\mathcal{C} = (\mathbb{Z}^N, +, 0, \mathbb{N}_0^N)$	$\# \mathcal{M} = 1$	$\forall_{t \in T} \text{Pre}(t), \text{Post}(t) \in \mathbb{N}_0^{2N}$
Coloured Petri nets $N$ places, $k$ colours	$\mathcal{C} = (Q^N, *, 1_N, \mathbb{N}^N)$ $Q$ pos. rat. numbers	$\mathcal{M} = \{p_1, p_2, \dots, p_k\}$ $p_\kappa$ suitable primes	$\forall_{t \in T} \forall_{\kappa=1}^k \text{Pre}(t, p_\kappa), \text{Post}(t, p_\kappa) \in \mathbb{N}^{2N}$
FIFO nets with $N$ places, alphabet $A$	$\mathcal{C} := (F_A^N, \cdot, \varepsilon, (A^*)^N)$ $F_A$ free group gener. by $A$	$\# \mathcal{M} = 1$	$\forall_{t \in T} \text{Pre}(t) \in A^* \times \{\varepsilon\}$ & $\text{Post}(t) \in \{\varepsilon\} \times A^*$
Continuous nets	$\mathcal{C} = (\mathbb{R}^N, +, 0, \mathbb{R}_0^N)$	$\mathcal{M} = \mathbb{R} \setminus \{0\}$	$\forall_{t \in T} \text{Pre}(t, \mu) = \mu \cdot \text{Pre}'(t)$ & $\text{Post}(t, \mu) = \mu \cdot \text{Post}'(t)$ $\forall_{t \in T} \text{Pre}'(t), \text{Post}'(t) \in \mathbb{R}^{2N}$

In the talk the following items will be considered in more detail:

- Homomorphisms of Petri nets, sub-Petri nets,
- Petri nets with an abelian cone group,
- S-invariants, T-invariants,
- Petri nets with sides-conditions.

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\*e-mail: BBrosowski@t-online.de